Reflections on Pile Dynamics

Bengt H. Fellenius

Keynote paper to the 5th International Conference on the Application of Stress-Wave Theory to Piles Orlando, Florida, September 1996

Fellenius, B.H., 1996. Reflections on pile dynamics. Keynote paper to the 5th International Conference on the Application of Stress-Wave Theory to Piles, September 10 through 13, Orlando, Florida, Proceedings pages Keynote 1 - Keynote 15.

Reflections on Pile Dynamics

Bengt H. Fellenius[1]

Keynote paper to the 5th International Conference
on the Application of Stress-Wave Theory to Piles
Orlando, Florida, September 1996

Almost all design analysis in foundation engineering originated from observations by practitioners in the field and the theoretical "background" was established as explanation to the observations. It is a paradox that pile driving, of all foundation practices the one most intimately associated with field behavior, saw substantial theoretical development well before solid field tests were performed (Isaacs, 1931, Smith, 1960). Well, dynamic measurements require electronic instrumentation and ability to record short duration events, which was not possible to do reliably until the mid and late 1950's. From then on, along with the development of the understanding of the fundamentals, grew the realization that determining what actually occurred is vital to the advancement of the piling technology.

In the expansive period after W.W.II in Sweden, the then head of the Swedish State Railways Geotechnical Department, my father, Bror Fellenius, tried to measure the forces occurring during a hammer impact. In 1948, he used a device consisting of three nickel spheres sandwiched between two smooth steel plates. When the device was subjected to an axial static load, each sphere gave a lasting circular impression on the steel plates and the diameters of these impressions were calibrated to force. The device was placed on a pile head and the impression from letting a hammer impact the device were used to determine the impact force (Fellenius, 1969). A series of measurements showed that the impact forces were smaller than previously assumed, but, of course, no information was obtained on the dynamics of the impact. A few years later, in 1956, when the Swedish Railways expanded the Stockholm Central Station, the need came up again to learn what actually happens during the driving of a pile. The piling contractor proposed to use light pneumatic hammers instead of conventional drop hammers to drive a large number of steel piles. Realizing that pile driving with pneumatic hammers must have a good deal in common with percussion drilling, Bror Fellenius contacted Atlas Copco, a company marketing large size drilling equipment and met with Dr. Hans Christian Fischer, who had done research using strain gages to determine dynamic forces in drill rods. The testing on the steel piles gave very interesting results and Dr. Fischer could demonstrate, using his technique of graphodynamic representation of stress waves (Fischer, 1960), the effect of various hammers on the magnitude of the reflected stress wave and provide a rational for the hammer selection and termination criteria.

[1)] University of Ottawa, Ottawa, Ontario, K1N 6N5

Bror Fellenius both realized the potential of the new measurement technique and the need for the people with the special know-how to get together with those of the many different disciplines involved in the finished product: the construction of the pile, the pile driving, the analysis of test data obtained from short duration events, the equipment manufacturers; in short, the contractors, the engineers, and the authorities. He invited representatives from all fields to a meeting and the outcome was the birth of the Swedish Pile Driving Committee in 1959 (later appointed to the Royal Academy of the Engineering Sciences under the name of the Swedish Pile Commission). The stated purpose of the Pile Committee was to advance the knowledge of piling through the sharing of information and research collaboration between the all parties involved. Almost immediately, an opportunity came about for a full-scale research programme. The Highway Authority and the Railways were constructing a major highway and railroad interchange including numerous bridges in a place called Gubbero ("*Old Man's Rest*") in the southwestern city of Göteborg where the soil consisted of some 60 m (200 ft) of soft clay over sand. The piles for the project were 270 mm (11 in) diameter hexagonal precast concrete piles made up of 10 m (30 ft) segments spliced during the driving. At the time, almost nothing factual was known about the behavior of the piles under these conditions. However, the membership of the newly formed Pile Committee included all the persons and organizations necessary to implement a research programme. The Highways and the Railroad provided much of the money and generous contributions were received from the industry, in particular from one of the piling contractors led by Mr. Sölve Severinsson. Mr. Severinsson was an innovative pioneer—a visionary—in the development of piling techniques in Sweden and an initiator of new techniques, such as the track-mounted crawler rig, reliable splices for precast concrete piles, rock shoes, and much more. He also became one of the prime supporters of the Pile Commission and is owed much of the credit for its success, then and later.

The Gubbero testing programme was carried out during 1960 and 1962 and consisted of the driving of a group of ten about 70 m long, instrumented piles with drop hammers, single-acting diesel hammers, and pneumatic hammers. The piles were equipped with strain gages (glued to the reinforcing bars at different levels) for measuring the dynamic behavior (strain waves) and with center pipes for inclinometer measurements of location and bending. The programme also included a study of splice performance, comparison of hammer and pile cushions of different stiffness and type, determining pile capacity and resistance distribution during short and long-term static loading tests, and logging of the penetration resistance in different soil layers as a function of nominal hammer energy, as well as the development of driving and termination criteria for the piles to be used to support the Gubbero bridges.

The testing programme gave a massive amount of information. It is not a fault of the investigation that many more questions came out of the results than were answered. In a sense, the Gubbero tests taught more about how not to do things than how to do them. Most important, the Gubbero tests became the impetus not just to an advancement of Swedish piling technology, but it established the precedent for collaboration between the parties involved in industry as well as authorities and researchers. This I later learned was also one of my father's primary goals in initiating the Pile Committee in the first place. When he in 1971 left the chairman position of the, as then called, Pile Commission, it had become a model institution for how to work out a consensus and collaboration in a potentially very adversary field. The Commission received international recognition, serving, for example, as an inspiration at the start of the Deep Foundation Institute in 1974 (Fellenius, 1994), and it is still a very active body.

I remember the Gubbero tests well, because in the Summer of 1960 I was a civil engineering student working on the project. While much of the project was a first for everyone involved, all of it was a first for me. Although, I admit that I feel more an awe for the undertaking now that I have experience which that ignorant student did not have. Indeed, the Gubbero test was a major step in everybody's education on pile behavior. The report (Fellenius et al., 1964) was written in Swedish and is therefore not accessible to more than a few (although it does have a comprehensive English summary). The Report deserves a wider audience, though, and I will highlight a few of the results.

In the 1950's, strain gage measurements of short-duration events were recorded by an oscillograph giving a trace on a screen. A permanent record was obtained by means of photographing the screen and determining the pertinent values by a hand-held ruler. Fig. 1 shows an example of force-time wave traces obtained in the Gubbero tests. Compression is downward and the scale is given by the grid system. The example is taken from Fig. 107 in the Gubbero Report and shows the force traces from two contiguous impacts recorded by a strain gage placed 20 m down into a 75 m long pile. The measurement is from the end of initial driving when the pile penetration resistance was 12 blows/25 mm for a 2,800 kg drop hammer impacting a well-used pile cushion.

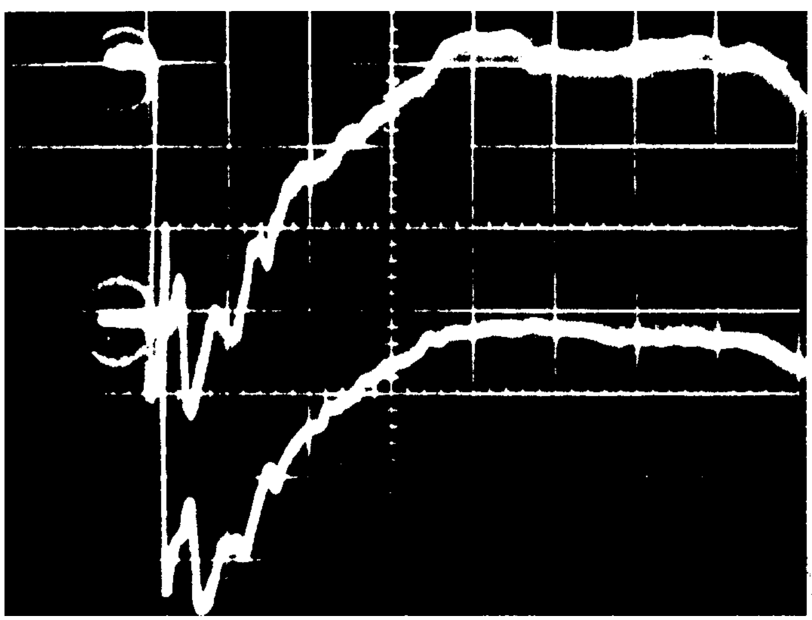

Fig. 1. Two contiguous force wave traces from the Gubbero tests

To report the measurements, the photographs were traced by hand onto a transparent paper that could serve as an original for copy making. (Copies were "blue print"; a photo copier was a rare piece of equipment in most offices at the time). Fig. 2 shows a set of force traces demonstrating the effect of different hammers in driving the piles. The force scale ("*Kraftskala*") is 33 tons per division, (323 KN – 73 kips) and the time scale ("*Tidskala*") is 10 ms per division. The example is taken from Fig. 105 in the Gubbero Report.

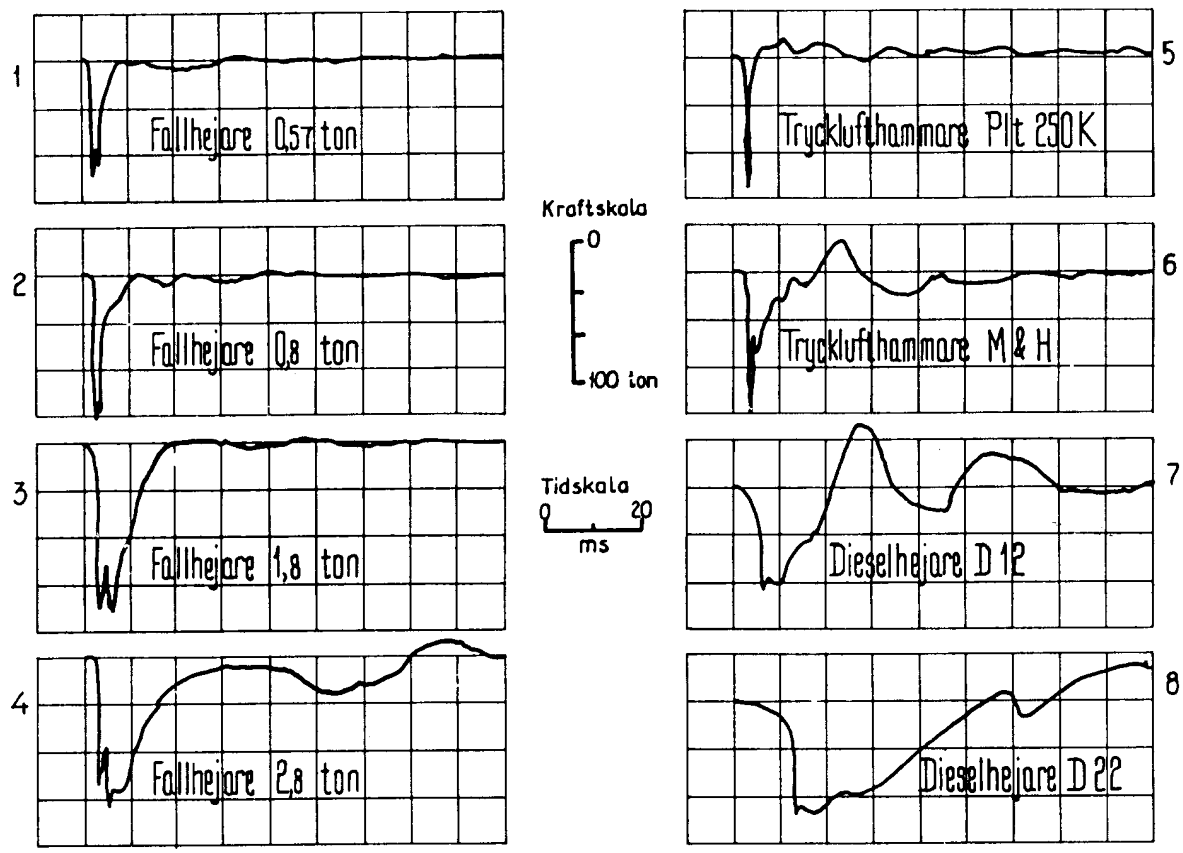

Fig. 2. Force wave-traces from the Gubbero tests
"*Fallhejare*": Drop hammer; "*Tryckluftshejare*": Pneumatic hammer; "*Dieselhejare*": Diesel hammer

The Gubbero Report addresses the maximum impact force, the "shape" of the trace, and the duration of the impact. The latter is discussed without consideration of the fact that the force trace is affected by the reflected wave — reduced if the reflection is in tension and increased if it is in compression. However, the upper about 50 m to 60 m length of the soil profile consisted of soft sensitive clay that was remolded and could generate a reflected force wave (wave-up) of no significant magnitude to add to the downward wave. Therefore, at a wave speed of about 3,300 m/s, no significant reflection would have affected the traces before about 3 divisions into the records. (the report does not contain any information as to from which piles and what their actual lengths were when the Fig. 2 traces were obtained).

The strain gage records were used to obtain information on the soil response, notably compression during hard driving and tension during easy driving. For compression, it was clear that the maximum values would occur near the pile head and at the pile toe. These were therefore the obvious places for the gages when studying compression. Fig. 3 presents records from one pile taken when the pile toe was at 57.5 m, 58.6 m, and 61.5 m below the ground surface, The pile was driven with a 2,800-kg drop hammer, a Delmag D22 diesel hammer, and again the drop hammer, respectively. Notice the time offset of about 20 ms for the traces at the pile toe. (The diagram is a detail of Fig. 75 in the Gubbero Report). A 'striking' visual 'impact' (not shown in Fig. 3, though) was achieved by presenting in the same figure a diagram over the penetration resistance versus depth.

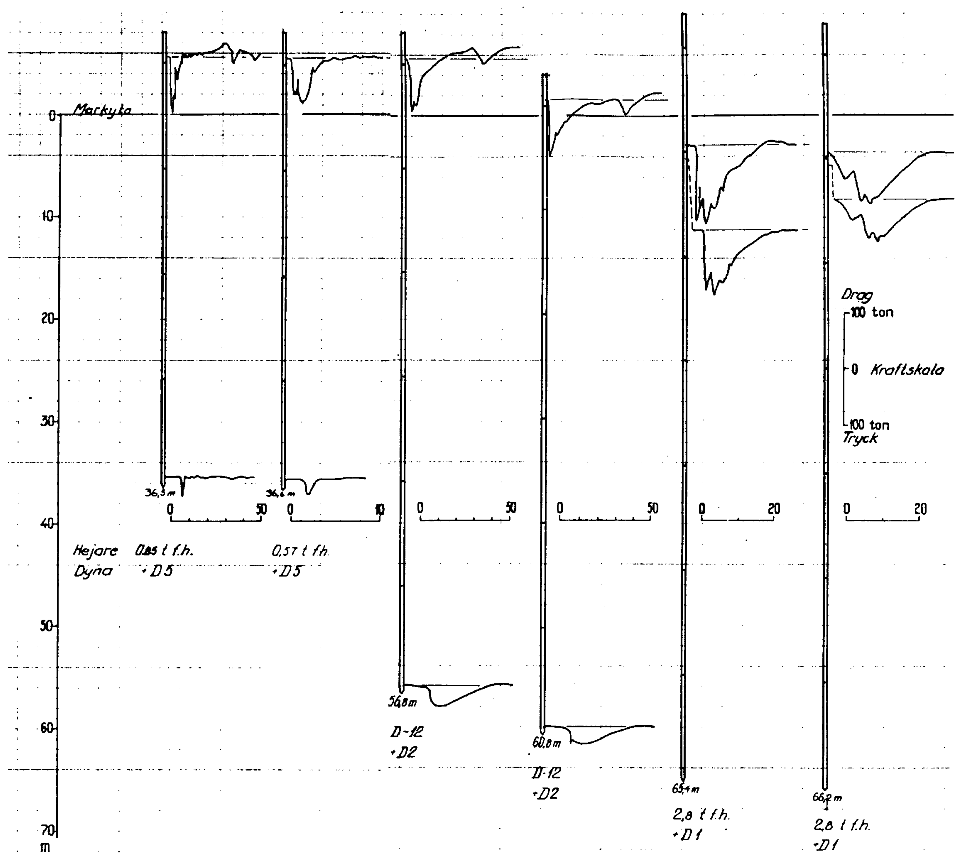

Fig. 3. Pile ET force wave-traces
"*Tidskala*" = Time scale; "*Kraftskala*" = Force scale; "100 ton" = 220 kips
"*Hejare*" = Drop hammer; "*Dyna*" = Cushion; "0.85 *t f.h.*" = 850-kg drop hammer
"*Tryck* = Compression; "*Drag*" = Tension; "*Markyta*" = Ground surface

At the depth of about 35 m, where the pile toe was still in clay, the penetration per blow was about 20 mm. Clearly, the pile must be subjected to considerable driving tension. However, the gage near the pile head is a "free end" and cannot register any tension. In contrast, the records shown in Fig. 4, where the gages are located at about halfway down the pile, indicate tension in the pile. (Fig. 4 is a detail of Fig. 70 in the Gubbero Report).

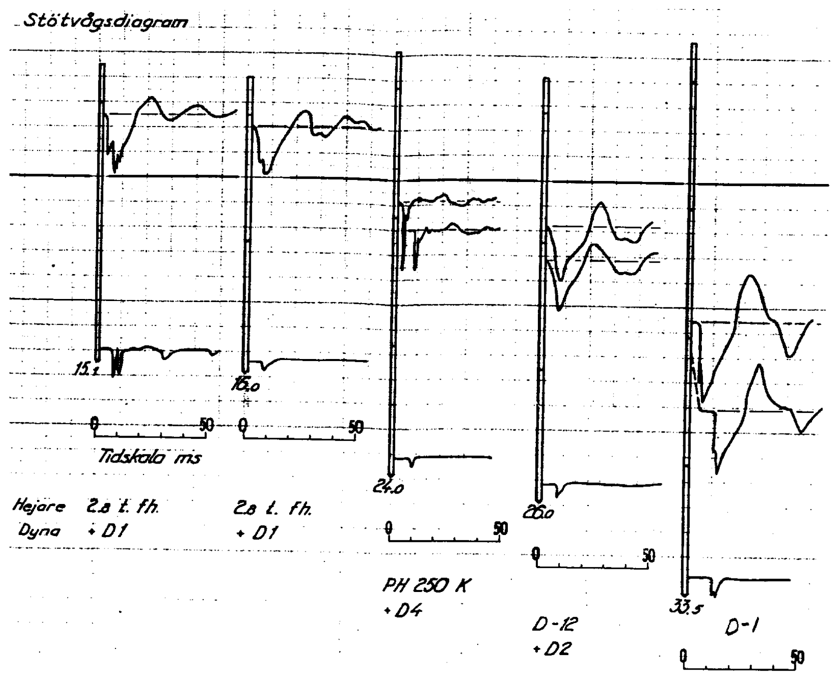

Fig. 4. Pile BT force wave-traces

Fig. 5 shows the results from measurements on a pile having strain gages both at the pile head and near the mid-point of the pile. The tension reflection is clearly evident in the record from the mid-point gage, but not visible by the gage near the pile head. (The diagram is a detail of Fig. 78 in the Gubbero Report).

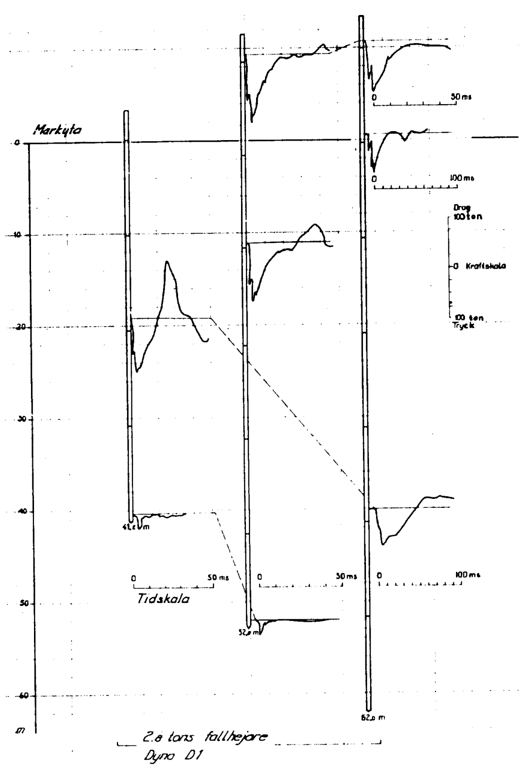

Fig. 5. Pile P2T force wave-traces

One obvious conclusion from these Gubbero results was that for a study of tension forces, which can be critical for a long concrete pile, the gages cannot be near the pile head, but should be somewhere in the middle of the pile. This made a study of driving tension in concrete piles rather cumbersome and expensive.

Both compression and tension forces are obtained from the measured strain multiplied by the section E-modulus of the pile and the pile cross sectional area. As is discussed in the report, this is fine for compression, but can be questionable for tension. If the pile would be cracked right at the gage location, tension exists only in the reinforcing bar (the strain-gage was glued to the reinforcement) and the indicated tension would only be a tenth of the value for an uncracked section of the pile. Direct measurement of tension strain can therefore be very misleading.

As capacity was not considered possible to determine from the dynamic measurements, a series of static loading tests were performed to verify that the hammers had succeeded to drive the piles to a sufficient capacity in the sand layer. Fig. 6 presents the results from the static loading test on Pile P1T.

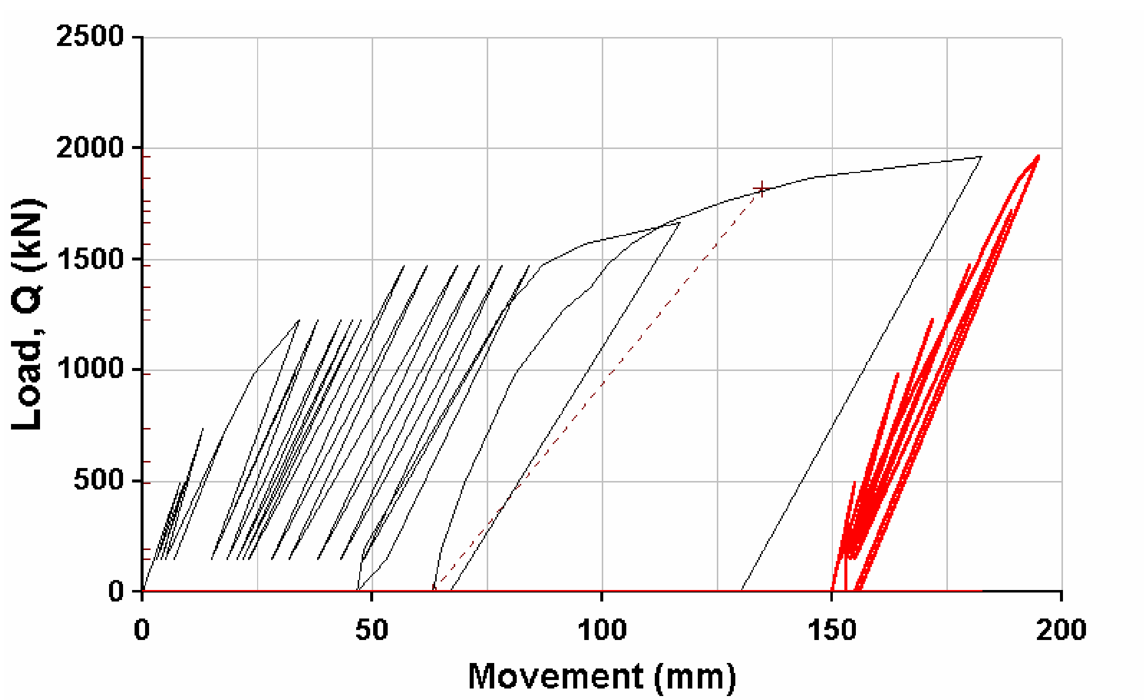

Fig. 6. Static loading test on Pile P1T (plotted from data in the Gubbero report)

Static loading tests on Pile P1T were performed on two occasions. First, in November 1960 and, then, in July 1962, 5 months and 24 months after the end of the driving, respectively. In Sweden at the time, static testing was normally by a time-consuming cyclic method (described by Fellenius, 1975) consisting of numerous cycles between a low load (150 KN in this case) and successively larger loads (only a selection of the load cycle measurements is shown in Fig. 6). On both occasions, the loading test was finished off with a quick-test consisting of adding constant increments of 100 KN until failure or maximum available reaction capacity was reached. The diagram shows that the pile head gross and net movement for the maximum load of 1,960 KN (220 tons -- 440 kips) in the November 1960 tests was about 180 mm and 130 mm, respectively. The thick lines show the load-movement results of the July 1962 test (for reasons of clarity, the curve is offset by 20 mm to start from 150 mm). In comparing the two test results, it is obvious that while failure was reached in the first test, this is not the case for the second test. It is probable that some of the pore pressures induced in the driving of the <u>pile group</u> still remained after five months, but they would have dissipated after 24 months. Fellenius (1969) presented measurements of pore pressure dissipation with time after driving for single piles driving in the same general soil at Bäckebol, a few kilometre away from the Gubbero test site and found that the dissipation time was about five months for a <u>single</u> pile.

The dashed line shown in Fig. 6 is the elastic line rising from the beginning of the concluding quick-test curve. The Gubbero Report defined its intersection with the curve as the failure load. Judging from the slope of the unloading curve, it appears that the line is not steep enough, that is, the E-modulus was underestimated. Correcting the slope and adding an offset (for these piles, the offset according to the Davisson Offset Limit is 6 mm), results in about the same value. Clearly, the failure load lies somewhere closer to the maximum load applied.

Without going into further details, the test results shown in Fig. 5 and the results from static loading tests applying similar method of testing on two more of the piles driven at the test site indicated that the traditional cyclic testing method gave almost no information of value for the interpretation of the pile behavior that a quick loading test could not provide at a fraction of the costs and time. Within a few years after the Gubbero tests, consequently, the Swedish industry had shifted to quick testing methods that include no unloading cycles, defining the failure load from the shape of the load-movement curve and not by a specific movement value.

The analysis from the static testing and static analyses included in the report did not consider negative skin friction or residual loads. That problem did not become a research objective until many years later.

The Swedish Pile Commission continued to perform pile research. However, for the next dozen years or so it accomplished only little in the field of pile dynamics. The further dynamic development was to take place outside Sweden, notably, in the USA. I can still remember the rush of excitement when attending the Second Case Seminar in the Spring of 1973 arranged by Dr. Goble and waking up to that the question was not to measure ***either*** with strain gages ***or*** accelerometers, but with ***both***. Until then, as far as my well-informed opinion went, either gage could be used and the gages shared the same difficulty: if either gage were placed at a free end, such as at the pile head, when the wave reflected from the soil resistance arrived to the gage location, game was over. This, because force determined from a strain gage either increased (compression wave reflection) or increased (tension wave reflection), while force determined from an accelerometer (integrated to velocity and multiplied with pile impedance) either decreased or increased, and, therefore, neither could separate the hammer force input from the reflected force. For study of tension, the gages had to be placed in the middle of the pile, but, then, they were not reusable and had to be written off in the test, which made pile dynamics research costly. Besides, the tension actually observed was not the net tension as the downward traveling wave would still be affecting the measurements.

At the Second Case Seminar in Cleveland, Dr. George Goble demonstrated that when having both gage types and placing them at the pile head, the fact that the force and velocity records have opposing trends (i. e., the strain gage and accelerometer responses on arrival of the soil reflections to the pile head) could be used to "tell it all", because the separation of the traces actually 'reflects' the dynamic soil resistance along the pile shaft. The origin of involving both types of gages in the test was based on the idea that the strain gage would give the force and the accelerometer tell when the pile velocity was zero. The force in the pile at zero velocity must be the force without damping, that is, the pile capacity. Well, it was not quite that simple, but the gage combination was there and its significance was quickly realized. The two independent measurements gave, qualitatively, a visual picture of the distribution of shaft resistance along the pile and a good picture of whether or not there was significant toe resistance. Records from initial driving and restriking obtained when both gage types were placed at the pile head could now give a clear indication of soil set-up, for example.

These days, the benefit of combining the gages is so clear that one might wonder why it was not obvious long before, say, in Gubbero. However, it only is obvious when the two traces are plotted to the same scale. You know, there are still those who do not do this! Or worse, who report dynamic measurements without providing a single wave trace.

Fig. 7 shows an example of force and velocity traces obtained from initial driving of a pile and from restriking the pile after the soil has had time to "set up", that is, recover from the driving disturbance. The difference in distance between the two traces in each pair is representative for in the gain of shaft resistance.

The message can be made even more evident by showing the "wave-down" and wave-up" traces. The wave-down is the average of the force and velocity, that is, the reflected wave has been taken away—remember, when soil resistance reflections occur, the strain gage and the accelerometer gages react in opposite directions. The wave-up is the half of the difference between the force and velocity traces and it then, shows the soil response to the blow as illustrated in Fig. 8. The solid line is the wave-down trace and this curve is less important than the dashed curve, the wave-up trace that 'reflects' the soil resistance. The wave-up trace clearly indicates that there has been quite an increase in the soil resistance between the end-of-initial-driving and restriking events.

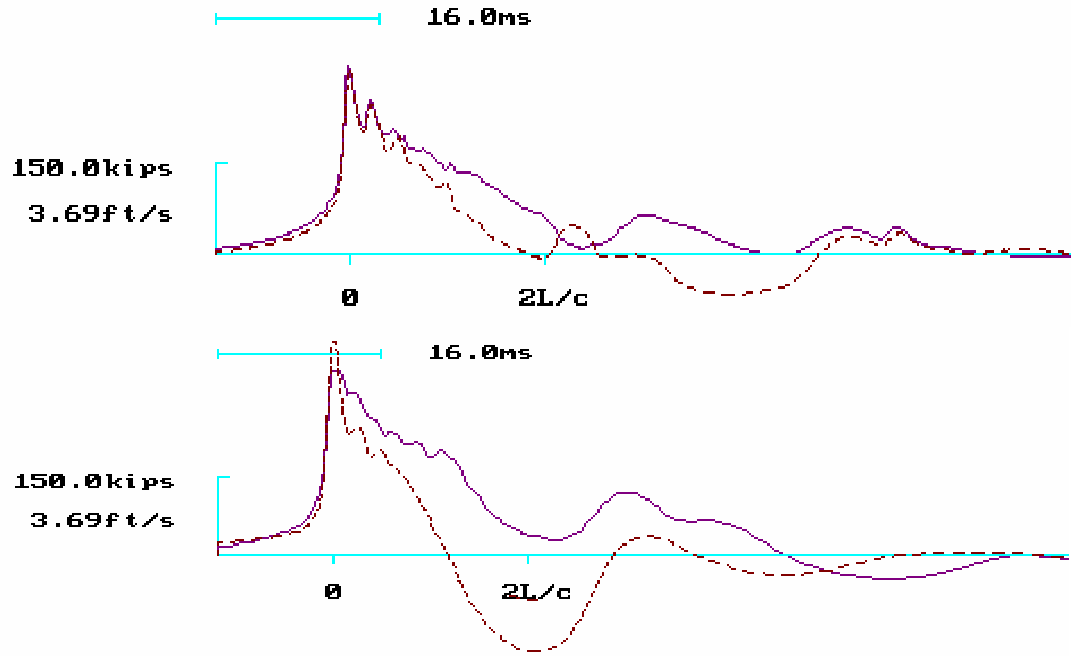

Fig. 7. Force and velocity wave traces obtained at end of driving and at restrike

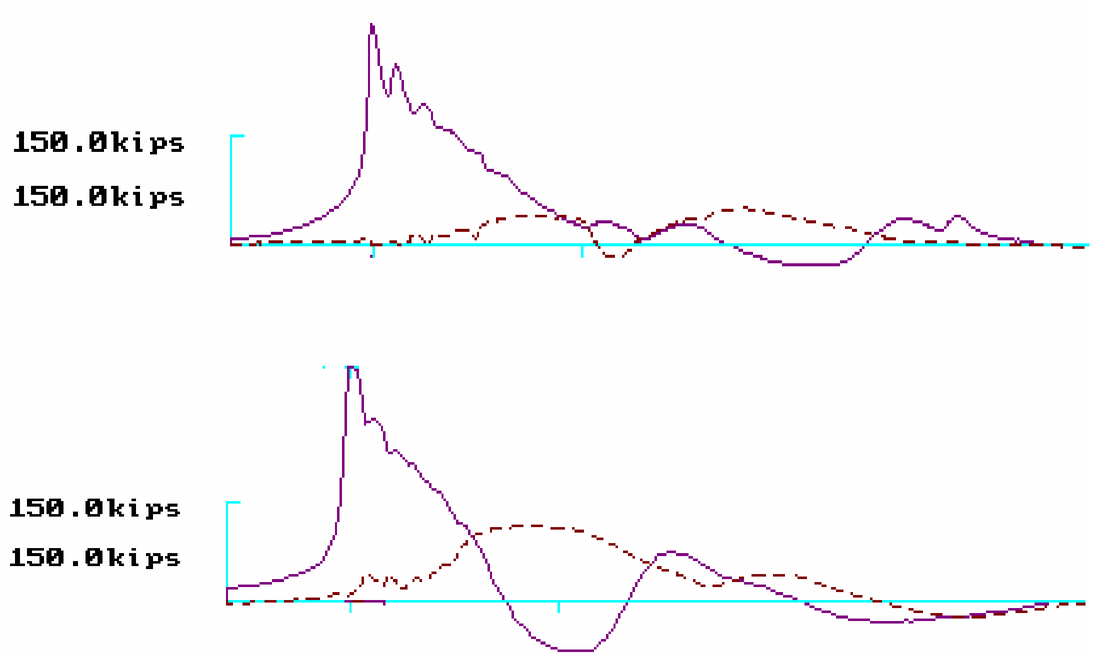

Fig. 8. Wave-down and wave-up traces for the case in Fig. 7

The Second Case Seminar introduced the participants to a device, then called, “Pile Capacity Computer”, later to be called “Pile Driving Analyzer”. I will leave to George to tell why the name was changed from “Computer” to “Analyzer”. The reason for changing from “Capacity” to “Driving” should be obvious, though, because the PDA does much more than determining capacity. As a matter of fact, on many projects, information on the impact force, the energy transferred to the pile, and the consistency of these data are more important and valuable than the pile capacity. Fig. 9 shows the current presentation of PDA data where the continued record is compiled to facilitate a evaluation of the piling.

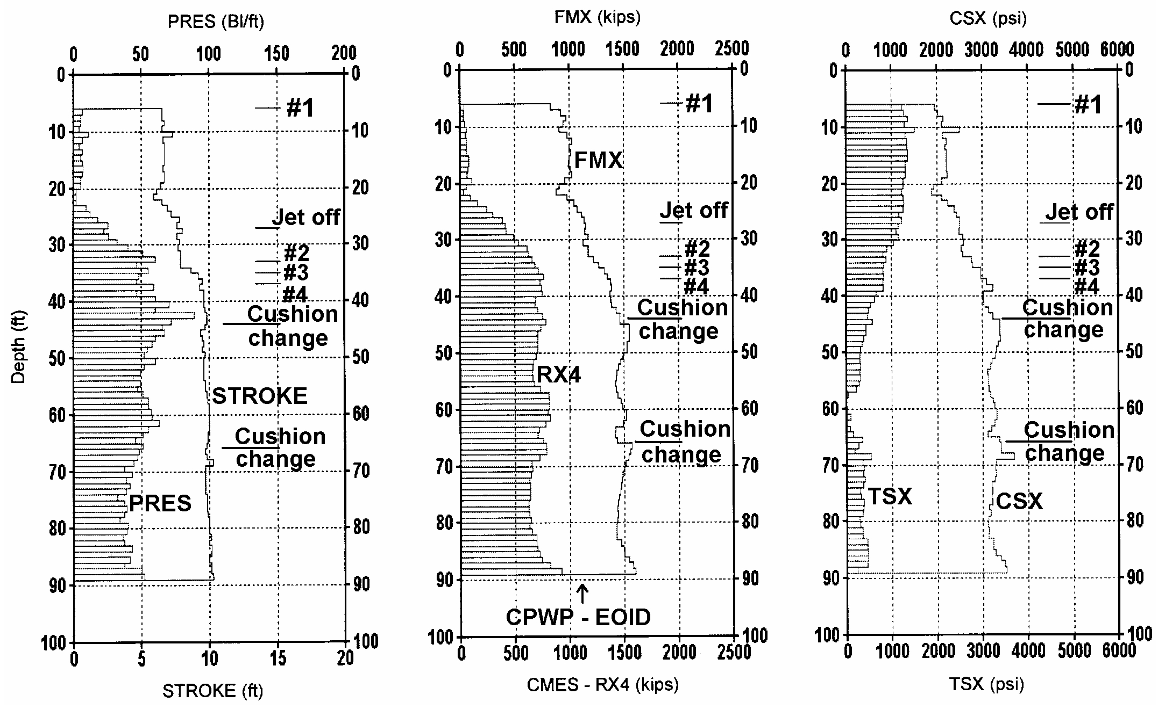

Fig. 9. Pile driving diagram from the Pile Driving Analyzer

At that same Case Seminar, I was also introduced to the CAPWAP analysis by Dr. Frank Rausche. Of course, while I realized that the CAPWAP would give a quantified value of capacity for the pile monitored by means of the two gage types, I think I was already so awed over being able to get a subjective impression of the magnitude of soil resistance and where it occurred that much of the prominence of the CAPWAP went past me. More about CAPWAP later, though. First, I must tell about another related experience.

In 1975, I got involved in the preliminary designs of two projects, one in Montreal and one in Hawaii. Both involved precast concrete piles and I was fortunate to be able to enlist the services of Francois Tavenas at Laval University to run wave equation analyses using the Texas A&M University main-frame program. I knew better than to depend on only a single analysis for any one case, but run enough calculations to produce upper and lower boundaries of everything. (Incidentally, this is still a very valid insight). Those were the days of the punch cards, and for a few weeks, I think, enough cards were punched and enough runs made with massive amount of fan-folded print-outs of results for me to support a good portion of the Canadian paper industry. Until that time, I had relied on my trusted slide rule—which, incidentally, I still trust, still carries, and still use—and, on occasion, a Hewlett Packard desk-top programmable calculator. When the Third Case Seminar was held in Cleveland in 1974 or 1975, the topic was the WEAP which Frank Rausche introduced. At the seminar workshop session, after three hours of intellectual and mechanical hard work, I had punched a few cards, almost without any help and was ready for my life's first computer run. There were about 15 participants in the workshop, and all fifteen runs were made from one day to the next. How fast it was!

The WEAP was a public domain program having been produced on a grant from the FHWA. Those who wanted the program received it in a shoe box filled with cards including one or more cards having holes initialized for the computer that the program was to run on. While paying nothing for the program itself, the effort of producing the cards was charged at $200, equivalent to about $1,000 today. And it only took the computer technician about a day to install it. The efficiency and ease of the thing! Note, I am not ironic. By the way, I know I was far from the first to obtain the WEAP program, but I must have been the only one, or at least one of very few, who paid the $200 twice in the same week for the same installation: This because I happened to drop the first shoe box on the floor—upside down. Carrying a punch-card box is like smoking a pipe made of chalk, you do not pick it up for ffurther use once it has fallen to the floor.

Much later, in 1985, I got a copy of the WEAP for the IBM PC and became independent of the main-frame. In those days — only eleven years ago! — it was necessary to spend a few extra minutes to think through the input, because the length of time for one run was enough to go down to the cafeteria for a refill, and, if the input was not the correct one, one had too much coffee in a day.

Then late in 1975, I got involved in a project which required dynamic testing with the Pile Driving Analyzer and I finally got to experience what the CAPWAP really is. To have sat next to Frank listening to his arguing with the computer why the input must have been just right and then hearing the renewed somewhat stronger argument when the computer disagrees, to, finally, experiencing the elation of seeing the “perfect” match roll off the graphics plotter from right to left and upside down and sharing the celebration of the success and one of Frank's cigars is a memory I cherish. It was also an invaluable training in understanding what actually goes on between a pile and the soil with the hammer as the match maker or with the hammer sometimes being not so good a match.

In 1976, I got my hands on my first Pile Driving Analyzer, PDA. That's the one I actually got to learn to use in the field. Along with the PDA becoming more and more like a computer and easier to handle, I have much to my regret become less and less versed in the actual taking of the measurements. But it has not held me back, because during these past 20 years, I have been fortunate to be able to rely and work with some excellent engineers — too many to single out the individuals — who have been much more adept with the PDA than I.

The progress of the dynamic field since that Summer in Gubbero has been enormous. I have progressed from mistake to mistake, most of which I have been able to learn from. Back in 1977, for example, working with Dave Thompson in Timmins, Ontario, we obtained some strange looking PDA data. The traces showed both large tension and distinct compression toe reflection in the pile for the same blow. Capacity seemed to be OK, meaning close to the desired value, if we applied a rather small J-factor. The question was, of course, whether the J-factor to apply really should be small in the silty clayey glacial soil at the site. A CAPWAP was requested from Cleveland and simultaneously a static loading test was performed. The static loading test showed a much smaller capacity that we had expected considering the hard driving and the hammer that was used to drive the piles. After all, previous experience with the hammer in similar soils had proven that for that final blow count, the capacity should have been more than enough. The CAPWAP results arrived about the same day that the test was completed and showed a good agreement with the static loading test. To get the traces to match, Frank Rausche had had to adjust the program to accept a quake several times larger than the until then preordained tenth-of-an-inch value. This was the first experience with a large toe quake, now so commonly observed. Jean Authier and I presented the case history to the First Stress-Wave Conference in Stockholm and showed that the ability of a hammer to drive a pile to a certain capacity reduces considerably if the quake increases (Authier and Fellenius, 1980).

Incidentally, the First Stress-Wave Conference was a really pleasant and informative three-day affair. There were many international groups reporting results from their research in dynamic testing and analysis. On the first day, much debate was heard why one system or the other was superior to the next. On the second day, most had found a common reference as measure to their work, that is, the test results and experiences of George Goble and co-workers. On the third day, George looked rather puzzled at times—no one was giving him an argument any more.

I know of no other geotechnical field where theory is so intimately combined with full-scale field measurements and used directly in the design of everyday project. The industry have benefited much from the information presented at the previous meetings and, I am sure, we will benefit not less from the Fifth Conference. However, let all the new sophistication make us lose sight of the basics. As a reminder of what sometimes goes on out there in the field and what unfortunate level of ignorance we can meet, I close with the following simple case history.

At a site up North, which the contract specifications characterized as 40 feet (12 m) of dense sand on a thick deposit of clayey silt, a group of 23 steel piles were to be driven to an embedment of 85 feet. The penetration resistance was considerably larger than the contractor had anticipated in the bid, but the ‘engineers’ held him to his contract. Fig 10 illustrated the blow-count he experienced in order to reach the specified embedment. The figure also indicates the N-values from the borehole which was stated to be from the site. In the supposedly clayey silt layer below 40 ft, the penetration resistance ranged, erratically, from 200 bl/ft thorough higher than 1,200 bl/ft. I do not think that I need to tell you that driving for more than a foot at 200 bl/ft is normally considered to be excessive.

Indeed, the 200 bl/ft value is often termed “practical refusal”. Notice, in this case, the contractor met that resistance at about 40 ft and had to drive an additional 45 ft. Actually, while the heads of the about 87 ft long piles eventually reached the intended level of 2 feet above ground, nobody knows where the pile toes went. To make a long story short, in process of the subsequent litigation for an $8 million claim, it was disclosed that the borehole was not from the particular site. That’s not the worst of it, nor that the contractor could not refuse to continue driving (or he had forfeited his bond). The worst of it is that no one had enough knowledge to understand what is happening and stop the insanity. In the midst of our advancing the state-of-the-art, we should consider that some of the players in our field have been left behind. We have an obligation to include also the pile driving basics in our dissertations.

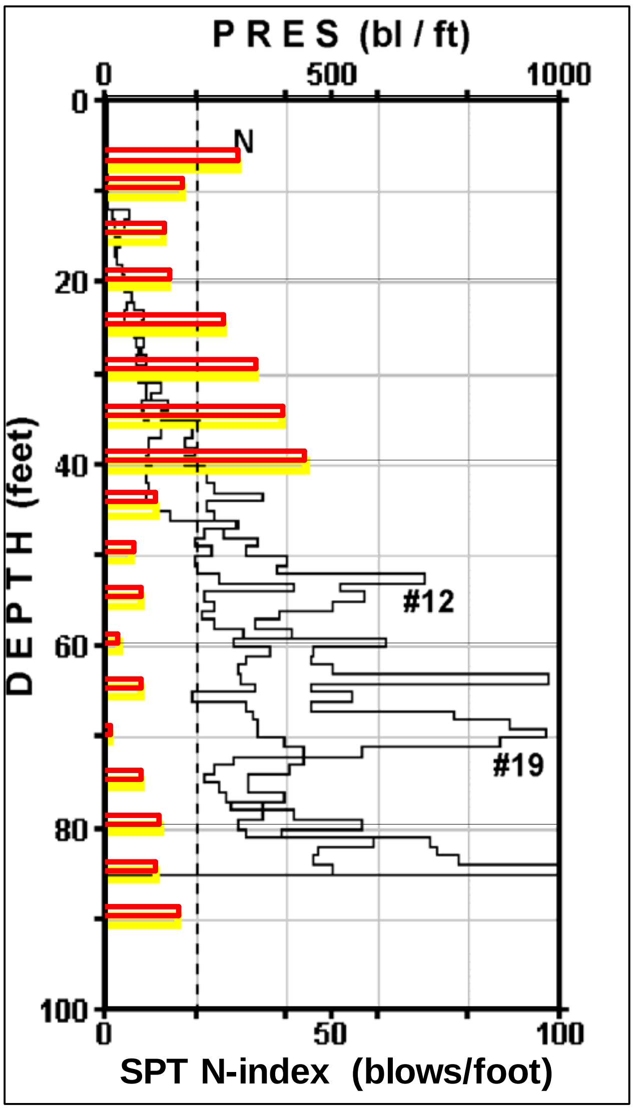

Fig. 10. Penetration resistance of two piles and SPT N-indices

References

Authier, J. and Fellenius, B. H., 1980. Quake values determined from dynamic measurements. Proceedings of the 1st International Seminar on the Application of Stress-wave Theory to Piles, Stockholm, 1980, H. Bredenberg, Editor, A. A. Balkema, Rotterdam, pp. 197 - 216.

Fellenius, B. G. W., Ahlberg, P. Fischer, H. C., Pejrud, W., Stål, T., and Sahlin, S., 1964. Slagning och provbelastning av långa provpålar vid Gubbero i Göteborg. AB Svensk Byggtjänst, Stockholm, 230 p. (In Swedish).

Fellenius, B. G. W., 1969. The development of piling technology. A retrospective. Väg och Vattenbyggaren, Vol. 15, pp. 3 - 7. (In Swedish).

Fellenius, B. H. and Broms, B. B., 1969. Negative skin friction for long piles driven in clay. Proceedings of the. 8th ICSMFE, Mexico, Vol. 2, pp. 93 - 97.

Fellenius, B. H., 1972. Downdrag on piles in clay due to negative skin friction. Canadian Geotechnical Journal, Vol. 9, No. 4, pp. 323 - 337.

Fellenius, B. H., 1975. Test loading of piles. Methods, interpretation, and proof testing. American Society of Civil Engineers, ASCE Journal of the Geotechnical Engineering Division, Vol. 101, GT9, pp. 855 - 869.

Fellenius, B. H., 1994. A look back at the beginnings of the DFI. Fulcrum, Deep Foundations Institute, April 1994, p. 4.

Fischer, H. C, 1960. On longitudinal impact, I - II. The Hague Martinus Nijkoff. Also in Tryckluft, Atlas Copco AB, No. 1, 1961, and NO. 2, 1962.

Isaacs, D. V. 1931. Reinforced concrete pile formulae. Transactions of the Institute of Engineers, Australia, Vol. 12, Paper No. 370, pp. 325 - 323.

Smith, E. A. L., 1960. Pile driving analysis by the wave equation. American Society of Civil Engineers, ASCE Journal of Mechanical Engineering,, SM4, pp. 35 - 61.